Pre-Algebra - Properties of Algebra

Objective: Simplify algebraic expressions by substituting given values, distributing, and combining like terms

In algebra we will often need to simplify an expression to make it easier to use. There are three basic forms of simplifying which we will review here.

World View Note: The term "Algebra" comes from the Arabic word al-jabr which means "reunion". It was first used in Iraq in 830 AD by Mohammad ibn-Musa al-Khwarizmi.

The first form of simplifying expressions is used when we know what number each variable in the expression represents. If we know what they represent we can replace each variable with the equivalent number and simplify what remains using order of operations.

Example 1.

$p(q+6)$ when $p=3$ and $q=5$	Replace p with 3 and q with 5
$(3)\mathbf{((5)+6)}$	Evaluate parenthesis
$\mathbf{(3)(11)}$	Multiply
33	Our Solution

Whenever a variable is replaced with something, we will put the new number inside a set of parenthesis. Notice the 3 and 5 in the previous example are in parenthesis. This is to preserve operations that are sometimes lost in a simple replacement. Sometimes the parenthesis won't make a difference, but it is a good habbit to always use them to prevent problems later.

Example 2.

$x+zx(3-z)\left(\frac{x}{3}\right)$ when $x=-6$ and $z=-2$	Replace all $x's$ with 6 and $z's$ with 2
$(-6)+(-2)(-6)\mathbf{(3-(-2))}\left(\frac{\mathbf{(-6)}}{\mathbf{3}}\right)$	Evaluate parenthesis
$-6+\mathbf{(-2)(-6)}(5)(-2)$	Multiply left to right
$-6+\mathbf{12(5)}(-2)$	Multiply left to right
$-6+\mathbf{60(-2)}$	Multiply
$\mathbf{-6-120}$	Subtract
-126	Our Solution

It will be more common in our study of algebra that we do not know the value of the variables. In this case, we will have to simplify what we can and leave the variables in our final solution. One way we can simplify expressions is to combine like terms. **Like terms** are terms where the variables match exactly (exponents included). Examples of like terms would be $3xy$ and $-7xy$ or $3a^2b$ and $8a^2b$ or -3 and 5. If we have like terms we are allowed to add (or subtract) the numbers in front of the variables, then keep the variables the same. This is shown in the following examples

Example 3.

$$\begin{array}{rl} 5x-2y-8x+7y & \text{Combine like terms } 5x-8x \text{ and } -2y+7y \\ -3x+5y & \text{Our Solution} \end{array}$$

Example 4.

$$\begin{array}{rl} 8x^2-3x+7-2x^2+4x-3 & \text{Combine like terms } 8x^2-2x^2 \text{ and } -3x+4x \text{ and } 7-3 \\ 6x^2+x+4 & \text{Our Solution} \end{array}$$

As we combine like terms we need to interpret subtraction signs as part of the following term. This means if we see a subtraction sign, we treat the following term like a negative term, the sign always stays with the term.

A final method to simplify is known as distributing. Often as we work with problems there will be a set of parenthesis that make solving a problem difficult, if not impossible. To get rid of these unwanted parenthesis we have the distributive property. Using this property we multiply the number in front of the parenthesis by each term inside of the parenthesis.

Distributive Property: $\boldsymbol{a(b+c)=\mathbf{ab}+\mathbf{ac}}$

Several examples of using the distributive property are given below.

Example 5.

$$\begin{array}{rl} 4(2x-7) & \text{Multiply each term by } 4 \\ 8x-28 & \text{Our Solution} \end{array}$$

Example 6.

$$\begin{array}{rl} -7(5x-6) & \text{Multiply each term by } -7 \\ -35+42 & \text{Our Solution} \end{array}$$

In the previous example we again use the fact that the sign goes with the number, this means we treat the -6 as a negative number, this gives $(-7)(-6)=42$, a positive number. The most common error in distributing is a sign error, be very careful with your signs!

It is possible to distribute just a negative through parenthesis. If we have a negative in front of parenthesis we can think of it like a -1 in front and distribute the -1 through. This is shown in the following example.

Example 7.

$$\begin{aligned} -(4x-5y+6) &\quad \text{Negative can be thought of as} -1 \\ -1(4x-5y+6) &\quad \text{Multiply each term by} -1 \\ -4x+5y-6 &\quad \text{Our Solution} \end{aligned}$$

Distributing through parenthesis and combining like terms can be combined into one problem. Order of operations tells us to multiply (distribute) first then add or subtract last (combine like terms). Thus we do each problem in two steps, distribute then combine.

Example 8.

$$\begin{aligned} 5+3(2x-4) &\quad \text{Distribute } 3\text{, multipling each term} \\ 5+6x-12 &\quad \text{Combine like terms } 5-12 \\ -7+6x &\quad \text{Our Solution} \end{aligned}$$

Example 9.

$$\begin{aligned} 3x-2(4x-5) &\quad \text{Distribute} -2\text{, multilpying each term} \\ 3x-8x+10 &\quad \text{Combine like terms } 3x-8x \\ -5x+10 &\quad \text{Our Solution} \end{aligned}$$

In the previous example we distributed -2, not just 2. This is because we will always treat subtraction like a negative sign that goes with the number after it. This makes a big difference when we multiply by the -5 inside the parenthesis, we now have a positive answer. Following are more involved examples of distributing and combining like terms.

Example 10.

$$\begin{aligned} 2(5x-8)-6(4x+3) &\quad \text{Distribute 2 into first parenthesis and} -6 \text{ into second} \\ 10x-16-24x-18 &\quad \text{Combine like terms } 10x-24x \text{ and } -16-18 \\ -14x-34 &\quad \text{Our Solution} \end{aligned}$$

Example 11.

$$\begin{aligned} 4(3x-8)-(2x-7) &\quad \text{Negative (subtract) in middle can be thought of as} -1 \\ 4(3x-8)-1(2x-7) &\quad \text{Distribute 4 into first parenthesis,} -1 \text{ into second} \\ 12x-32-2x+7 &\quad \text{Combine like terms } 12x-2x \text{ and } -32+7 \\ 10x-25 &\quad \text{Our Solution} \end{aligned}$$

0.4 Practice - Properties of Algebra

Evaluate each using the values given.

1) $p+1+q-m$; use $m=1$, $p=3$, $q=4$

2) y^2+y-z; use $y=5$, $z=1$

3) $p-\frac{pq}{6}$; use $p=6$ and $q=5$

4) $\frac{6+z-y}{3}$; use $y=1$, $z=4$

5) $c^2-(a-1)$; use $a=3$ and $c=5$

6) $x+6z-4y$; use $x=6$, $y=4$, $z=4$

7) $5j+\frac{kh}{2}$; use $h=5$, $j=4$, $k=2$

8) $5(b+a)+1+c$; use $a=2$, $b=6$, $c=5$

9) $\frac{4-(p-m)}{2}+q$; use $m=4$, $p=6$, $q=6$

10) $z+x-(1^2)^3$; use $x=5$, $z=4$

11) $m+n+m+\frac{n}{2}$; use $m=1$ and $n=2$

12) $3+z-1+y-1$; use $y=5$, $z=4$

13) $q-p-(q-1-3)$; use $p=3$, $q=6$

14) $p+(q-r)(6-p)$; use $p=6$, $q=5$, $r=5$

15) $y-[4-y-(z-x)]$; use $x=3$, $y=1$, $z=6$

16) $4z-(x+x-(z-z))$; use $x=3$, $z=2$

17) $k\times 3^2-(j+k)-5$; use $j=4$, $k=5$

18) $a^3(c^2-c)$; use $a=3$, $c=2$

19) $zx-(z-\frac{4+x}{6})$; use $x=2$, $z=6$

20) $5+qp+pq-q$; use $p=6$, $q=3$

Combine Like Terms

21) $r-9+10$

22) $-4x+2-4$

23) $n+n$

24) $4b+6+1+7b$

25) $8v+7v$

26) $-x+8x$

27) $-7x-2x$

28) $-7a-6+5$

29) $k-2+7$

30) $-8p+5p$

31) $x-10-6x+1$

32) $1-10n-10$

33) $m-2m$

34) $1-r-6$

35) $9n-1+n+4$

36) $-4b+9b$

Distribute

37) $-8(x-4)$

38) $3(8v+9)$

39) $8n(n+9)$

40) $-(-5+9a)$

41) $7k(-k+6)$

42) $10x(1+2x)$

43) $-6(1+6x)$

44) $-2(n+1)$

45) $8m(5-m)$

46) $-2p(9p-1)$

47) $-9x(4-x)$

48) $4(8n-2)$

49) $-9b(b-10)$

50) $-4(1+7r)$

51) $-8n(5+10n)$

52) $2x(8x-10)$

Simplify.

53) $9(b+10)+5b$

54) $4v-7(1-8v)$

55) $-3x(1-4x)-4x^2$

56) $-8x+9(-9x+9)$

57) $-4k^2-8k\,(8k+1)$

58) $-9-10(1+9a)$

59) $1-7(5+7p)$

60) $-10(x-2)-3$

61) $-10-4(n-5)$

62) $-6(5-m)+3m$

63) $4(x+7)+8(x+4)$

64) $-2r(1+4r)+8r(-r+4)$

65) $-8(n+6)-8n(n+8)$

66) $9(6b+5)-4b(b+3)$

67) $7(7+3v)+10(3-10v)$

68) $-7(4x-6)+2(10x-10)$

69) $2n(-10n+5)-7(6-10n)$

70) $-3(4+a)+6a(9a+10)$

71) $5(1-6k)+10(k-8)$

72) $-7(4x+3)-10(10x+10)$

73) $(8n^2-3n)-(5+4n^2)$

74) $(7x^2-3)-(5x^2+6x)$

75) $(5p-6)+(1-p)$

76) $(3x^2-x)-(7-8x)$

77) $(2-4v^2)+(3v^2+2v)$

78) $(2b-8)+(b-7b^2)$

79) $(4-2k^2)+(8-2k^2)$

80) $(7a^2+7a)-(6a^2+4a)$

81) $(x^2-8)+(2x^2-7)$

82) $(3-7n^2)+(6n^2+3)$

Answers - Properties of Algebra

1) 7
2) 29
3) 1
4) 3
5) 23
6) 14
7) 25
8) 46
9) 7
10) 8
11) 5
12) 10
13) 1
14) 6
15) 1
16) 2
17) 36
18) 54
19) 7
20) 38
21) $r+1$
22) $-4x-2$
23) $2n$
24) $11b+7$
25) $15v$
26) $7x$
27) $-9x$
28) $-7a-1$
29) $k+5$
30) $-3p$
31) $-5x-9$
32) $-9-10n$
33) $-m$
34) $-5-r$
35) $10n+3$
36) $5b$
37) $-8x+32$
38) $24v+27$
39) $8n^2+72n$
40) $5-9a$
41) $-7k^2+42k$
42) $10x+20x^2$
43) $-6-36x$
44) $-2n-2$
45) $40m-8m^2$
46) $-18p^2+2p$
47) $-36x+9x^2$
48) $32n-8$
49) $-9b^2+90b$
50) $-4-28r$
51) $-40n-80n^2$
52) $16x^2-20x$
53) $14b+90$
54) $60v-7$
55) $-3x+8x^2$
56) $-89x+81$
57) $-68k^2-8k$
58) $-19-90a$
59) $-34-49p$
60) $-10x+17$
61) $10-4n$
62) $-30+9m$
63) $12x+60$
64) $30r-16r^2$
65) $-72n-48-8n^2$
66) $-42b-45-4b^2$
67) $79-79v$
68) $-8x+22$
69) $-20n^2+80n-42$
70) $-12+57a+54a^2$
71) $-75-20k$
72) $-128x-121$
73) $4n^2-3n-5$
74) $2x^2-6x-3$
75) $4p-5$
76) $3x^2+7x-7$
77) $-v^2+2v+2$
78) $-7b^2+3b-8$
79) $-4k^2+12$
80) a^2+3a
81) $3x^2-15$
82) $-n^2+6$